Chapter 3.4 The Concatenation Question

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Abstract
Gene tree discordance is now recognized as a major source of biological heterogeneity. How to deal with this heterogeneity is an unsolved problem, as the accurate inference of individual gene tree topologies is difficult. One solution has been to simply concatenate all of the data together, ignoring the underlying heterogeneity. Another approach infers gene tree topologies separately and combines the individual estimates in order to explicitly model this heterogeneity. Here we discuss the advantages and disadvantages of both approaches—using the gene trees singly or in concatenation—paying special attention to the sources of variance and implicit assumptions. We make it clear that all methods are likely to have their assumptions violated, though the consequence of these violations differs in different parts of parameter space. The main conclusion of our review is that different sources of error are more or less important in different settings, such that phylogenetics researchers should be using the methods most appropriate to their problems rather than stick to one dogmatically.

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1 Introduction

Phylogenetic inference is a hard problem, especially for deep divergences. There is the computational challenge: genomic data sets are huge and require sophisticated optimization and sampling algorithms. There is the statistical challenge: trees are awkward objects to do statistics on, and yet careful quantification of uncertainty is becoming more and more critical. There is the modelling challenge: access to full genome sequences has given us an appreciation of the complexity of evolutionary processes (see Chapter 2.1 [Simion et al. 2020]). Substitution rates vary across loci, across sites, across lineages and over time. Even the underlying Markov process can change (Inagaki et al., 2004; Jejeoy et al., 2006; Philippe et al., 2005, 2017).

The realization that different genes evolve under different substitution processes sparked the total evidence versus consensus debate in the 1990s (e.g. Page, 1996). The core point of disagreement was whether data should be analysed all together (total evidence) or separately and then combined (consensus; see Bull et al., 1993; De Queiroz, 1993). The debate spurred the development of new consensus methods, quartet methods, and tests for homogeneity. After a decade of sometimes bitter wrangling, the total evidence versus consensus debate died a natural death. The rise of efficient maximum likelihood and Bayesian software (see
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Chapters 1.2 and 1.4 [Stamatakis and Kozlov 2020; Lartillot 2020]), and improved computers, made combined analysis with heterogeneous models tractable, placating both camps.

Heterogeneity in the substitution process is one thing; heterogeneity in tree topology is another. The fact that different loci can have different underlying histories has been recognised for a long time (Hudson, 1983; Tajima, 1983; Pamilo and Nei, 1988). However, it was not until coalescent theory leaked from population genetics into systematics (Maddison, 1997) that many realised how ubiquitous gene tree discordance could be. We can now be confident that much of the topological variation in recent divergences we see among loci is due to biological causes rather than technical errors (e.g. Brawand et al., 2014; Heliconius Genome Consortium, 2012; Fontaine et al., 2015; Novikova et al., 2016; Pease et al., 2016; Pollard et al., 2006; Rogers et al., 2019).

The heightened awareness of gene tree discordance appears to have revived the total evidence versus consensus debate, albeit in a different guise. Those on the (new) consensus side, armed with the multispecies coalescent, argue that gene trees should be inferred separately and then combined to infer a species tree (e.g. Edwards, 2009). Those on the (new) total evidence side pick holes in these arguments and remind readers of older arguments in support of concatenation (e.g. Gatesy and Springer, 2014).

If history repeats itself again, the revived debate will die a natural death. New, improved model-based methodologies will eventually address the concerns of both camps. In the interim, the debate motivates the discussion of fundamental issues related to the scale and scope of gene tree discordance, the importance of statistical consistency, and relative sources of error in phylogenetic inference. It also provides a convenient framework for a chapter discussing those issues.

In Section 2 we introduce much of the relevant theory about the population processes that lead to discordance, discussing incomplete lineage sorting, expected branch lengths, the “anomaly zone”, and the impact of recombination. In Section 3 we discuss summary tree methods, approaches that estimate a gene tree for each locus and then combine these estimates. We discuss the consistency, and inconsistency, of these methods, and suggest that chopping the genome up into small chunks for separate analyses might open up these methods to systematic error and biases.

In Section 4 we examine the approach of concatenating all genes together before analysis, effectively ignoring the potential discordance among trees. This wholesale concatenation has been proven to be statistically inconsistent (Roch and Steel, 2015), though, as we point out, the branch lengths used in the proof are ridiculously short. The question of whether or not to concatenate is a question of finding a compromise between different kinds of error. The error from discordance exists for both recent and deep divergences, though with deep divergences it appears that alternative sources of error become much more important.

Section 5 examines one of the most confusing threads in the debate: whether or not we should tolerate recombination within loci. Oddly, both sides accuse the other of ignoring recombination and discordance. We discuss the arguments for and against combining trees, and some of the ways that have been proposed to overcome the bias associated with short loci. We also consider the pitfalls when considering clustered or binned genes as single loci in the multispecies coalescent.

In the final section we examine alternatives to summary methods and concatenated maximum likelihood, while also noting that there are excellent reasons for estimating individual gene trees, independent of species tree inference.
One of the most important findings from genome-scale data in phylogenomics is that gene tree discordance is ubiquitous. Recognizing discordance between gene trees—and accounting for it in the inference of species trees—has been a major focus of the last decade of phylogenetic methods development. Among all of the possible causes of discordance, incomplete lineage sorting (ILS) has received the most attention, though introgression between species may well have a comparable real impact (Mallet et al., 2016). Here we focus on ILS, a concept we introduce in Section 2.2. While gene duplication and subsequent loss is also often included as a biological cause of discordance (e.g. Degnan and Rosenberg, 2009; Maddison, 1997), it is due to the mis-assignment of paralogs as orthologs (see Chapter 2.4 [Fernández et al. 2020]), and we do not consider it further.

2.1 Basic coalescent thinking

Two randomly chosen sequences at a locus from a single population share a common ancestor in the recent past. Under the Wright-Fisher model of diploid, hermaphroditic organisms with effective population size $N$, the probability that two autosomal sequences sampled from a single generation find a common ancestor in the previous generation is $1/2N$. We use $2N$ here because each of the $N$ individuals carries two copies of this locus; alternatively, we can imagine a population of haploid individuals of size $2N$. A simple outcome of Mendelian inheritance is that the distribution of times back until two lineages find a common ancestor—that is, until they “coalesce”—is exponentially distributed with a mean of $2N$ generations. This process has a large variance, and independent loci sampled from the same two individuals will coalesce at many different times in the past.

Results for samples of size $n > 2$ can be derived under the $n$-coalescent model (Hudson, 1983; Kingman, 1982; Tajima, 1983). With $n = 3$ there are three equally probable topologies relating three lineages within a single population, and with $n = 4$ there are 18 equally probable labeled histories (by “labeled history” we mean that we distinguish between trees with the same relationships but that have lineages coalescing in a different temporal order). For all such topologies the coalescent model provides expectations for the times to coalescence, which in turn also imply branch lengths upon which mutations can accumulate (see Hein et al., 2004 and Wakeley, 2009 for an overview).

Importantly, in the $n$-coalescent model we assume that all observed mutations are neutral. This assumption allows us to completely separate the genealogical process of coalescence from the process by which mutations occur in the sample history. Every locus in this model has an underlying gene tree, irrespective of whether we are able to determine what it is from the pattern of informative mutations—our ability to infer a tree is not a necessary condition for its existence. More complex coalescent models than those described here are available, some incorporating selection, and some with non-Wright-Fisher populations (e.g. Spence et al., 2016). The importance of such models for phylogenetics is a largely unexplored area.

2.2 Incomplete lineage sorting

The small but finite amount of time it takes lineages to coalesce has significant consequences for variation in gene tree topologies. One useful way to think about this phenomenon is to ask whether all of the sampled lineages in a population have found their common ancestor before some pre-specified time in the past. Avise et al. (1983) referred to the case where all lineages find their common ancestor as “lineage sorting”. Conversely, we now refer to
the case in which there are two or more lineages remaining as “incomplete lineage sorting” (ILS).

To be concrete, consider time measured in coalescent units, so that $T = t/2N$, where $t$ is the number of generations. Given exponentially distributed coalescence times, the probability of lineage sorting of two lineages by time $T$ in the past (i.e. the probability of 2 lineages going to 1 lineage) is:

$$P_{21}(T) = 1 - e^{-T}. \quad (1)$$

Likewise, the probability of incomplete lineage sorting (i.e. the probability of 2 lineages staying as 2 lineages) is:

$$P_{22}(T) = e^{-T}. \quad (2)$$

This result implies that only $\approx 63\%$ of loci will have coalesced by the mean expected time to coalescence ($2N$ generations, or 1 coalescent time unit), but that $95\%$ of loci will have coalesced by $6N$ generations in the past. If we consider a species with an effective population size of 100,000 and a generation time of 1 year, these numbers imply that it would take on average 600,000 years for 95% of loci to sort. Similar calculations for the probability of lineage sorting among more than two lineages can also be made (Tavaré, 1984).

Incomplete lineage sorting is important for phylogenetics because it implies that, even when two species share an ancestral branch, not every gene tree sampled from those species will also have that branch. Enumerating the probabilities of specific topologies and their associated branch lengths in the presence of ILS is the goal of the multispecies coalescent model, which we discuss next.

### 2.3 The frequency of different topologies under the multispecies coalescent

Because of the time required for lineage sorting, ancestral populations that existed between closely spaced speciation events become very important. The multispecies coalescent (MSC) model (Hudson, 1983; Pamilo and Nei, 1988; Tajima, 1983; Takahata and Nei, 1985; Takahata, 1989) recognizes that coalescence in ancestral populations can determine the frequency and branch lengths of different topologies, and attempts to quantify these measures. The MSC is limited in many ways—it does not include many processes that can be modeled in the general coalescent framework—but it does provide an important guide to the effects of ILS on gene tree discordance.

Imagine that we have sampled one (haploid) individual from each of three species, A, B, and C, and that the true relationship between the species is $(A, B, C)$ (see Figure 1). We would like to know the probability of sampling a gene tree that matches this topology if we collect data from a single locus. Discordance at a locus can occur if the lineages sampled from A and B do not coalesce in their common ancestral population, and instead one of them coalesces with the lineage from C in the population ancestral to all three species. Regardless of how long the tip branches are (because no coalescence between species can occur along them), the probability that A and B do not coalesce in the most recent shared ancestral population is given by Equation 2, with $T$ denoting the length of this internal branch. If there is no coalescence (i.e. if ILS occurs) then each of the three possible topologies are equally likely to occur in the common ancestral population of A, B, and C.

Under this model, the expected frequencies of the two discordant topologies are both (Hudson, 1983):

$$E[f_{((A,C),B)}] = E[f_{((B,C),A)}] = (1/3)e^{-T}. \quad (3)$$
Figure 1 Incomplete lineage sorting and gene tree discordance. (a) Complete lineage sorting, so that the gene tree is consistent with the species tree. (b)-(d) Incomplete lineage sorting, of which only the first gives a gene tree consistent with the species tree.

A concordant topology is also produced under ILS, at the same frequency as the two discordant topologies (Hudson, 1983).

A concordant topology must be produced if there is lineage sorting (with probability $1 - e^{-T}$), so the total frequency of concordant topologies is:

$$E[J((A,B),C)] = (1/3)e^{-T} + (1 - e^{-T}) = 1 - (2/3)e^{-T}. \quad (4)$$

We can see that there is more discordance with very small internal branch lengths (up to a maximum of 2/3 of all trees), and that at very long internal branch lengths there is essentially no discordance due to ILS. Following from the example given above, at $T = 6$ approximately 95% of loci will have sorted in the common ancestor of $A$ and $B$, and will therefore be concordant. Of the remaining 5%, 1/3 will also be concordant, with the other loci equally split between the two discordant topologies.

Similar calculations can be made for arbitrarily large numbers of lineages undergoing ILS (Degnan and Salter, 2005). With four taxa undergoing ILS, there are now two internal branches of any species tree that must be considered, with ILS occurring in either one or both branches. While there are 18 possible labeled histories with four taxa, often only the 15 unlabeled histories are considered (e.g. Rosenberg, 2002), as we do not distinguish between, for instance, the two different possible sequences of coalescences in the topology $((A,B),(C,D))$ (either $(A,B)$ first or $(C,D)$ first). The number of possible topologies quickly explodes with more taxa. It is essential to realize, however, that these calculations reflect the number of lineages undergoing ILS, not the number of taxa in a tree. It may be that even in a tree of 100 taxa only 3 lineages are in a phylogenetic “knot” that induces ILS. In such cases we need only concern ourselves with ILS calculations for three taxa.

One of the most important take-home messages about the MSC is that ILS can occur at any time in the past. As can be seen from Equations 3 and 4, the only parameter determining the probability of discordance due to ILS is $T$, which measures the length of an
internal branch of the species tree in coalescent units (though note that most species trees are reported using absolute time or numbers of mutations per site per branch). Whether this internal branch existed 1 million or 100 million years ago, the amount of discordance due to ILS will be the same. However, our ability to determine a gene tree topology, and to ascribe it to ILS or not, is certainly dependent on how long ago these events occurred and how long the internal branches of individual gene trees are.

2.4 Gene tree branch lengths under the MSC

The expected branch lengths in both concordant and discordant gene trees are easily obtainable from the MSC model. As the coalescent process can have no effect on tip branch lengths after the most recent speciation event when one sequence is sampled from each species, total branch lengths will always have tip branch lengths added as a constant. Therefore, we focus on the expected lengths of gene tree branches above the tips.

When ILS occurs among three lineages, two coalescent events occur in the common ancestral population. Looking backwards, the first coalescence is expected to occur $2N/3$ generations in the past (Figure 1(b-d)). Following this event, which is equally likely to join any of the three possible pairs of sequences, there are two lineages left in the tree, and therefore an average of $2N$ more generations until the entire sample has reached its most recent common ancestor. This result means that the internal branch of any topology that is the result of ILS (whether concordant or discordant) has an expected length of $2N$ generations. Even in “hard polytomies”, where the length of the internal branch of the species tree is zero, each of the three gene trees has an internal branch with expected length $2N$. In contrast, the internal branch of loci that complete the lineage sorting process have a minimum expected length of $2N$ generations and a maximum length equal to the length of the internal branch of the species tree. Mendes and Hahn (2018) provide analytical formulas for these expectations.

To give some perspective on how long such an internal branch is, recall that the expected number of pairwise differences between two sequences within a population is $4N\mu$, where $\mu$ is the per-nucleotide mutation rate (often this compound parameter is referred to as $\theta$). The expected number of mutations on the internal branch of discordant trees is $2N\mu$, or about half the number of pairwise differences. As the proportion of sites with such differences are generally at or below 1% for multicellular organisms (Leffler et al. 2012), we can begin to understand why it is so hard to accurately identify discordant gene trees. Note that this internal branch of discordant trees has the same short length no matter how far back in time the ILS occurred and no matter how long the internal branch of the species tree is. This is why discordant gene trees due to ILS will always have lower bootstrap support than any concordant tree—because they always have a shorter internal branch—and why using a bootstrap cut-off to determine which gene trees to include in an analysis could result in a biased estimate of discordance.

2.5 The anomalous anomaly zone

Under the multispecies coalescent, each species tree determines a distribution on the set of gene trees. There have been several theoretical results on this distribution, mainly for the case where exactly one individual is sampled from each species. It is tempting to think of this distribution as a cloud of random gene trees, centred on the species tree. Degnan and Rosenberg (2006) showed that this picture can be misleading. One can construct a species tree such that the most likely gene tree under the multispecies coalescent is not the species
tree. This observation, perhaps more than any other, has been used to justify methods that account for incomplete lineage sorting in inferring the species tree.

Consider the species tree in Figure 2. If the two successive internal branches in this species tree are short enough (in coalescent units), coalescences are most likely to occur in the ancestral population of all four lineages. Under the coalescent model in a single population, symmetric trees, like ((A,B),(C,D)) have higher probability than asymmetric (“caterpillar”) trees like (((A,B),C),D) because the former are associated with two labeled histories while the latter are associated with only a single labeled history. As a result, if the species tree is a caterpillar and most coalescence occurs in the single ancestral population, an asymmetric gene tree matching the species tree can be less common than one of the symmetric gene trees. The term “anomaly zone” is used to describe the area of parameter space (in terms of branch lengths in the species tree), where the gene tree concordant with the species tree is less probable than some other tree.

Despite the anomaly zone’s bogeyman-like status in phylogenetics, it is not as scary as it looks. For instance, there is no anomaly zone for gene trees with branch lengths. The times between coalescent events have an exponential density. The density function of an exponential random variable is strictly decreasing, so gets larger for values close to zero. The mode, or most “likely” value, for an exponential random variable is zero, and the most likely time of coalescence is zero or effectively instantaneous. As a consequence, the mode of the distribution of gene trees with branch lengths under the MSC is identical to the species tree! For the same reason, in the multispecies coalescent with extremely small population sizes (such that pairs of lineages coalesce as soon as possible), every gene tree would match the species tree exactly.

Nevertheless the anomaly zone does cause problems with methods for inferring species trees based on counts or frequencies of gene trees. It was also thought that the anomaly zone was responsible for consistency problems with concatenated maximum likelihood, though as we will see later it is not the anomaly zone that is responsible.
2.6 The coalescent with recombination

The classical MSC model makes two important assumptions about the role of recombination, neither of which is likely to be true in real data but both of which are required to produce the topological distributions expected under the MSC. The first assumption is that we are dealing with individually non-recombining loci, such that each locus or gene contains only a single underlying topology. Non-recombining loci such as mtDNA, cpDNA, or the Y (or W) chromosome all conform to this assumption. For sequences drawn from the autosomal nuclear genome, the length of non-recombining loci is a function of rates of recombination and population sizes \( (N) \).

These considerations raise the question of how long we expect non-recombining loci in the nuclear genome to be. The rate of recombination varies along the genome and across species. In humans, the average length of non-recombining autosomal loci is 4.8-5.9 kilobases (International HapMap Consortium, 2005), though there is a huge variance in the length of such blocks. For species with larger population sizes such as \textit{Drosophila}, there is more effective recombination and block sizes are commensurately smaller, on the order of hundreds of bases or less (Hey and Nielsen, 2004). However, because the amount of nucleotide diversity also scales with population size, the large block sizes in species with small populations like humans do not necessarily result in more phylogenetic resolution within each locus.

While a strict interpretation of the MSC assumes non-recombining loci, there are some kinds of recombinations that have no effect on inference. If the recombination is limited to lineages within a branch, different sites will have identical gene trees, even if they are undergoing recombination.

The real concern of intra-locus recombination for inference is when there are multiple histories present among loci, as when there is incomplete lineage sorting or introgression. When this occurs, different sites within a single protein-coding gene can have discordant gene trees. In fact, Mendes et al. (2019) showed that 70\% and 91\% of protein-coding genes in primates and \textit{Drosophila}, respectively, contain two or more gene tree topologies from a single phylogenetic knot (i.e. three species undergoing ILS). Even if genes were on average the same length as non-recombining stretches of chromosome, multiple trees will be combined unless the recombination events exactly flank the sequence being used for inference. When there are multiple knots across a larger tree, the length of loci that do not have a single recombination event within them at any point in the tree can become vanishingly small (Gatesy and Springer, 2014). We return to the issue of intra-locus recombination below, as it affects all of the methods we discuss here.

There is a second critical assumption that the MSC makes about recombination: that different loci have independent gene trees (conditional on the species tree). In other words, it assumes that there is sufficient recombination between loci that gene trees for different loci are independent (conditional on the species tree). Non-independence of samples is a common problem across statistics, known to cause greater variability than expected (overdispersion). The consequences of assuming independence in phylogenetics are not well understood, but generally assumptions of this type result in greater confidence in results than is warranted.

3 Summary gene tree methods

In a summary tree method, separate gene trees are estimated for each locus, and these gene trees are then used to infer the species tree. In the contemporary revival of the total evidence \textit{versus} consensus debate, those advocating summary tree methods fall squarely in the “consensus” camp. Examples of this approach include ASTRAL (Mirarab et al.,
D. Bryant and M. W. Hahn 2014), MP-EST (Liu et al., 2010), and ASTRID (Vachaspati and Warnow, 2015), with new methods and updates appearing monthly. Summary tree methods are sometimes referred to as “shortcut” coalescent methods, as the full likelihood of the species tree under the MSC is bypassed in favor of simple expedients.

3.1 Non-anomalous subtrees

Most existing summary tree methods depend heavily on a couple of key results regarding rooted triples and unrooted quartets of gene trees. In the anomaly zone the most probable gene tree under the MSC can be different than the species tree (ignoring branch lengths). Oddly enough, if we throw away taxa, the anomaly zone disappears. To demonstrate this phenomenon, consider the two following properties of the MSC:

1. In a rooted species tree with three taxa and one individual sampled per species, the most probable rooted gene tree is the same as the species tree, regardless of internal branch length.
2. In an unrooted species tree with four taxa and one individual sampled per species, the most probable unrooted gene tree is the same as the species tree, regardless of internal branch length.

Both of these observations are direct consequences of calculations from Tajima, Hudson, and Nei dating back 30-40 years; see Degnan and Rosenberg (2006) and Degnan (2013) for recent derivations.

At first glance, these properties of the MSC appear to create a paradox. If we consider all taxa at the same time, the most probable gene tree need not be the same as the species tree. However, if we consider every triple (in the rooted case) or quartet (in the unrooted case) then the most probable small trees will match the species tree, even though any rooted tree is determined by its triples and every unrooted tree is determined by its quartets. The explanation for this is that even the wrong trees might have some of the correct triples or quartets, and the probability of observing a particular triple combines the probability of observing it when the gene tree equals the species tree and the probability of observing it when it does not.

The absence of non-anomalous gene trees with four taxa (or three in the rooted case) makes it easy to design species tree methods that are consistent under the MSC. Given a sample of unrooted gene trees, we determine the most frequent four-taxon trees for each subset of four taxa. As the number of loci increases, the probability of inferring the four-taxon tree concordant with the species tree approaches one. Reconstructing the species tree from its quartets is straightforward. The case for rooted trees is the same, only there we deal with rooted triples (three-taxon trees) rather than quartets. The term “coalescent aware” was coined for methods which made use of these results, even though they do not necessarily require any coalescent calculations.

The realisation that methods using subtrees of larger trees are apparently immune to gene tree discordance led to a real 90s-style revival in phylogenetic methodology. A general approach that had, for the most part, fallen into disuse, suddenly became a mainstream tool in systematics and even required by some journal editors.

3.2 Inconsistency of summary tree methods

The concept of statistical consistency features prominently in the discussion and promotion of summary tree methods. In general, an estimator for a quantity is statistically consistent if the probability of it returning the correct value converges to one as the amount of data
increases, given that the data are generated by the assumed model. Thus a phylogenetic method is consistent if the method converges to the correct tree when there is no model violation and the amount of data (number of sites or loci) goes to infinity.

Current summary tree methods make the fundamental assumption that the inferred gene trees have the same distribution as do gene trees under the MSC. If the distribution is the same, methods based on rooted triples and unrooted quartets will infer the underlying species tree accurately, given sufficiently many loci. However in practice we do not have access to the gene trees themselves, but only estimates of the gene trees. Those estimates, particularly in deep phylogenetics, can be error prone.

For these reasons, Warnow (2015) describes two flavours of statistical consistency for the multispecies coalescent. The \textit{weak} version corresponds to consistency conditional on correctly inferred gene trees. The \textit{strong} version says that the estimated species tree will converge in probability as the number of loci increases even with a bound on the length of each locus. In practice, it is strong consistency that is relevant to phylogenetics.

Summary methods do not typically satisfy strong consistency. This is not that surprising—it would be a minor miracle if the distribution implied by the multispecies coalescent happened to line up exactly with the distribution resulting from inferring trees with sampling error. Roch et al. (2018) provide a formal proof of inconsistency. Here we will settle for an informal argument, one which illustrates a particularly important point. A similar phenomenon is documented by Wang et al. (2019).

Maximum likelihood (ML) is biased on finite sequences; it is, after all, a non-linear estimator. The most extensively studied example of bias in ML is “long branch attraction”, which describes the zone of parameter space in which long, non-sister branches are erroneously inferred to be sister (e.g. species \(A\) and \(D\) in Figure 3). This is widely known as the Felsenstein zone (Felsenstein, 1978). Though it is usually thought of as a larger stumbling block for parsimony, when the mix of branch lengths is sufficiently extreme, and sequences sufficiently short, the Felsenstein zone is also challenging for ML.

![Figure 3](image-url) Four taxon tree on which ML is biased with short sequences. Branch lengths are in expected percentage of non-identical sites (Adapted from Swofford et al., 2001).

Swofford et al. (2001) studied the problem of inferring trees simulated on the tree in Figure 3. The long branches have about 1.7 expected substitutions per site while the short branches have about 0.16 expected substitutions per site. If you simulate sequences of length 50 base pairs on this tree, then the maximum likelihood tree will be \(((A, D), (B, C))\) with probability 41\%, \(((A, B), (C, D))\) with probability 34\% and \(((A, C), (B, D))\) with probability 25\%. This implies that the maximum likelihood tree is correct only 34\% of the time.

Now suppose that the tree in Figure 3 is the species tree, and that population sizes are so small (or branch lengths so long) that there is negligible ILS. If the loci only have 50 sites each, then, as the number of loci increases, the frequency of the correct tree will converge
on 34%, while the frequency of the incorrect tree will converge on 41%. Any of the standard summary tree methods will then select the wrong tree as the species tree with certainty as the number of loci increases. In other words, summary tree methods are statistically inconsistent, in the “strong” sense.

A method can be statistically inconsistent and yet perform well in practice. In itself, a proof of consistency or inconsistency only tells us a limited amount because it only addresses asymptotic bias. In any statistical estimation problem, phylogenetic inference included, there is a trade-off between bias and variance. An estimator might have some bias associated with it, even with infinite data; however, it can be advantageous to just live with that bias if the overall level of error can be kept under control. The real value of proofs of inconsistency, or indeed of simulations demonstrating bias, is that they help us to identify contexts within which a method might be misleading. The classic proof of Felsenstein (1978) that parsimony is inconsistent is useful because it identifies important and real situations where the method can be misleading. It also helped to identify similar problem areas for other methods, including ML (e.g. Kim 1996).

3.3 The problem with summary tree methods

Summary tree methods can be inconsistent because maximum likelihood is biased. In the simple example we gave above (Figure 3), maximum likelihood would select an incorrect gene tree more often than the correct tree. The potential for long branch attraction, and other forms of bias, increases as phylogenies get deeper and the variation in evolutionary processes gets more complex. Variation in substitution rates among sites and across the tree make it difficult to correct for homoplasy and multiple substitutions.

Extensive work has been done examining these issues, and how they can affect phylogenetic inference (Philippe and Roure 2011; Philippe et al. 2011, 2017; Chapter 2.1 [Simion et al. 2020]). Not only can long branch attraction be difficult to diagnose, it can lead to a complete reshuffling of the inferred tree. This stands in contrast to ILS, which typically only has a local impact on the topology around short internal branches.

The standard strategy for dealing with heterogeneity in the substitution process is to try to construct generative models of how the processes can change. There are many advantages to using models, particularly the fact that we can start to understand features of the actual substitution process and their impact on inference.

Obviously, then, we would like to apply model-based approaches to the inference of gene trees. However, in order to fit complex models and to carry out reliable inference using these models, we need long sequences. We need longer sequences because modelling variation will inevitably result in increased sampling variance and small-sample bias. In the example of Figure 3 it took slightly more than 50 sites to overcome the bias. For larger, deeper trees, and multi-faceted, complex models, it could take many many more sites. Kück et al. (2012) report alignments where maximum likelihood is still biased with over 100,000 sites!

By chopping up the genome and analysing each fragment independently, summary tree methods run the risk of substantial and systematic bias, replicated independently for each locus. The obvious solution to this problem is to join loci together to make longer alignments, but this approach has pitfalls of its own. It is these problems that we discuss next.

4 Concatenation

Standing in the opposite corner from summary tree analysis stands the total evidence, or concatenation, approach. In this approach all loci are analysed together, using models that
incorporate complex substitution processes. We call this approach *concatenated ML*. The advantages of concatenated ML are that all of the models and technologies developed for deep phylogenetics can be applied to the concatenated alignment. The sequence lengths are long enough to fit the models with some semblance of reliability, which ensures more accurate inferences for large areas of parameter space.

### 4.1 Inconsistency of concatenated ML

The problem with concatenating all of the loci is that concatenated ML is not statistically consistent on data generated by the MSC, a fact that has been used repeatedly to justify summary gene tree methods. In general, if a parameter is identifiable, and you use a maximum likelihood estimator with the correct model for the data, then a maximum likelihood estimator is consistent. As a consequence, concatenated ML is consistent on data generated on a single gene tree. However, it is not consistent on data generated on discordant gene trees (or a mix of concordant and discordant trees). Given the existence of incomplete lineage sorting and the anomaly zone, it should perhaps come as no surprise that maximum likelihood has trouble on alignments containing many conflicting gene trees.

The existence of the anomaly zone does not, by itself, imply statistical inconsistency of concatenated ML under the MSC. Nevertheless, simulation studies (Kubatko and Degnan, 2007) indicate that the short internal branches of caterpillar trees associated with the anomaly zone cause problems for maximum likelihood estimation on concatenated data. Mendes and Hahn (2018) have shown that concatenated ML can fail both inside and outside the anomaly zone, as the reasons for the inconsistency are not directly driven by the identity of the most probable gene tree. Indeed, both parsimony and neighbor-joining appear to perform surprisingly well in the presence of large amounts of discordance, on species trees both inside and outside the four-taxon anomaly zone (Liu and Edwards, 2009; Mendes and Hahn, 2018).

The first analytical proof of the inconsistency of concatenated ML under the MSC was provided by Roch and Steel (2015). A nice overview of the proof can be found in Warnow (2015). As with the anomaly zone proof, the basic issue is that when internal branch lengths are very short almost all of the coalescent events occur in the ancestral population. Because gene trees generated under the coalescent model for a single population favour one kind of tree over others, the gene trees resulting from species trees with very short internal nodes also favor one (wrong) topology over the others (Figure 4).

![Figure 4](#) The species tree for which concatenated ML fails in Roch and Steel (2015). The times between speciation events are rather short: for effective population size $N = 100,000$, $\theta = 0.001$ and a generation time of one year, consecutive speciation events are separated by a little under 100 minutes.
To illustrate how short the internal branch lengths have to be for concatenated ML to fail in Roch and Steel’s proof, it is useful to consider the parameter values required by their construction. Suppose that there are \( n \) taxa, and let \( \beta \) be an upper bound on the length of any internal branches, measured in coalescent units. Claims 7 and 5 in Roch and Steel (2015) then imply that \( \beta \) satisfies

\[
\left( e^{-\left(\frac{n}{2}\right)\beta} \right)^n e^{-\theta(n^2\beta)} \geq 1 - \frac{\theta^2}{n}.
\]

If we plug in \( n = 6 \), which is the number of taxa used in their construction, we have \( \beta < 1.86 \times 10^{-9} \) for \( \theta = 0.001 \). To put this figure in context, suppose that we are considering a species with an effective population size of around 100,000 and generation time of 1 year. The number of generations along each internal branch is then bounded above by \( 1.86 \times 10^{-9} \times 100,000 \), corresponding to \( 1.86 \times 10^{-4} \) years, or just under 100 minutes. In other words, Roch and Steel have proven that concatenated ML is inconsistent on a species tree where the gap between divergences is less than 100 minutes, or 6-7 hours total for all five speciation events to occur.

We are being a little facetious here. The inconsistency result is correct and, moreover, we believe that concatenated ML will still continue to be inconsistent on more reasonable species trees (if difficult to prove analytically). However, there is also a serious side. Just because a method is inconsistent for some parameter values does not mean that it is not the best methods for others. We contend that this may well be the case for concatenated ML: when the amount of incomplete lineage sorting is small, perhaps relative to other sources of noise, concatenated ML performs extremely well (Mirarab et al., 2016). When the amount of ILS is large, then we should be concerned about the shortcomings of concatenated ML.

4.2 Balancing different sources of error

When we carry out concatenated ML analysis in the context of the MSC we are committing the sin of model misspecification: the model used for inference does not match the one that generated the data. Model misspecification is, of course, widespread in statistics—indeed there are few statistical analyses where model assumptions all hold exactly. The key issue is the extent to which model misspecification is misleading in practice and in context. Will the model misspecification completely rearrange the tree, or just locally distort the topology around a few tiny branches?

Context is particularly important. In phylogenetics different sources of error play quite different roles at different depths in the tree. Simulations, and theory, demonstrate that phylogenetic inference via concatenated ML is badly misled when branch lengths are very short, resulting in high levels of ILS. The error from ignoring ILS leads to local rearrangements in the tree, rather than global errors. Furthermore, in the area of parameter space where such errors occur, < 15% of all gene tree topologies match either the “true” species tree or the tree returned by concatenated ML (Kubatko and Degnan, 2007; Degnan and Rosenberg, 2006). In other words, no matter which tree is chosen, ~ 85% of loci in the genome will have a different evolutionary history. If rearrangements are local and all answers represent only a small minority of gene trees, then this source of error is far more benign than what we may expect from substitution rate errors and long branch attraction.

We can get a sense for the relative contribution of different sources of error by comparing the corresponding sources of variance when estimating genetic distances between species. Consider sequences sampled from two species separated \( t \) generations ago. Under a simple Poisson mutation model, the number of nucleotide differences between two sequences has
expectation

\[ 2t\mu + \theta \]  

and a variance of

\[ 2t\mu + \theta(1 + \theta) \]  

(Gillespie and Langley, 1979). Here \( 2t\mu \) is expected number of mutations since the time of species divergence, and \( \theta = 4N\mu \) is again the expected number of differences between two sequences sampled from the ancestral population at the time the lineages split.

The two terms in Equation 6 correspond to variance from the mutation process since divergence and variance from the coalescent process prior to divergence. How do these quantities compare in practice? A typical value for \( \theta \), and hence for the coalescent variance \( \approx \theta(1 + \theta) \), might be around \( \theta = 0.01 \). The value for mutational variance scales with \( t \), and therefore depends on how long ago the species split. If \( t \approx 2N \) then the two contributions to variance are almost identical: the variability from the coalescent process matches the variability from the mutational process. Hence the coalescent will be a major source of error. In a species with a generation time of one year and \( N = 100,000 \) this corresponds to a divergence time of 200,000 years.

As \( t \) increases the variance from mutation also increases, but the contribution from the coalescent stays the same. So at 1 million years, the variance of mutation will be five times that of the coalescent; at 10 million years it will be 50 times. Deeper than 20 million years, and the coalescent will contribute less than 1% of the variance. In effect, the misspecification resulting from ignoring the coalescent should have little or no impact on inference (at least for divergence times).

There are some obvious limitations in this example. For one thing, relative variances would change with more sequences, or more complex models. The key fact, though, is that mutational variance increases significantly with depth of divergence, whereas coalescence variance is the same at any depth. We expect coalescence to play a more significant role when unravelling recent divergences, but to be swamped by other sources of error when examining deeper divergences.

Two recent studies provide empirical evidence that mutational variance and the modelling error that comes along with it can dominate the inference of gene tree discordance in deep phylogenies. Richards et al. (2018) carried out a detailed and careful phylogenetic analysis of genes in the mitochondrial genome for several deep clades of vertebrates. Recombination in the mitochondria is rare, at least relative to autosomal recombination (White et al., 2013). Hence, any discordance observed among inferred trees is most likely to be a consequence of phylogenetic error rather than biological gene tree heterogeneity. Nevertheless, the study observed gene tree conflict at a level commensurate with that observed in nuclear genomes. While there was no “consistent” discordance, as there would be under ILS, the observation of strongly supported discordant trees is worrying.

Scornavacca and Galtier (2017) employed results on the expected length of internal branches of discordant trees to put an upper bound on the expected proportion of sites affected by ILS. Richards and colleagues (2014) carried out a detailed and careful phylogenetic analysis of genes in several deep clades of vertebrates. Recombination in the mitochondria is rare, at least relative to autosomal recombination (White et al., 2013). Hence, any discordance observed among inferred trees is most likely to be a consequence of phylogenetic error rather than biological gene tree heterogeneity. Nevertheless, the study observed gene tree conflict at a level commensurate with that observed in nuclear genomes. While there was no “consistent” discordance, as there would be under ILS, the observation of strongly supported discordant trees is worrying.

Scornavacca and Galtier (2017) employed results on the expected length of internal branches of discordant trees to put an upper bound on the expected proportion of sites affected by ILS. Using a rough estimate of \( \theta \) from extant mammals, they show that the observed proportion of sites supporting a gene tree discordant with the species tree is far higher than that expected. This result was stronger for deeper nodes in the placental mammal phylogeny, suggesting that only a small fraction of discordant sites can be explained by ILS deeper in the tree. In this context, other sources of error are swamping ILS.
In summary, then, there is no question that ILS is an important cause of gene tree discordance, especially when looking at recently diverged populations or species. It is not clear, however, that the phylogenetic “error” due to ILS trumps all other sources of error, especially as we move into the more distant past. It is not that problems due to ILS get smaller, only that the error due to all other causes gets much larger.

5 The role of recombination in the debate

5.1 Concatenation

One issue that we have mostly avoided discussing so far is whether the loci analyzed by summary tree methods are themselves non-recombining. A standard unit of phylogenetic analysis is the protein-coding gene. As mentioned earlier, the vast majority of protein-coding genes in eukaryotic genomes are likely to contain two or more topologies in the presence of ILS. While single exons less often contain multiple topologies (Mendes et al., 2019), they are much shorter and are therefore both less likely to be able to fully resolve trees containing many taxa and will provide many fewer sites with which to fit complex substitution models. The implicit compromise of using even single protein-coding genes is that we have enough sequence with which to carry out “good enough” phylogenetic analyses, even though we may be violating the MSC. This compromise approach has been given the portmanteau “concatenation” (Gatesy and Springer, 2014).

There has been quite a kerfuffle in the literature surrounding concatenation (Gatesy and Springer, 2014; Liu et al., 2014b; Springer and Gatesy, 2016; Edwards et al., 2016). It seems to us that the main question is not whether current approaches using single protein-coding genes violate the MSC—they almost certainly do—but what effect this violation has on the inferred topologies, and especially the distribution of inferred topologies.

We have already seen that in extreme cases of ILS, concatenated ML will converge on the wrong tree with more and more data (Kubatko and Degnan, 2007). What is less clear is the behavior of shorter genes that combine only a handful of different topologies. Some simulations have been done to examine the effect of recombination on realistic gene lengths (Lanier and Knowles, 2012), finding little effect relative to other sources of phylogenetic error. However, these simulations have been criticized as having too little recombination (because the intervening introns were not taken into account; Gatesy and Springer, 2014), and did not seem to include the areas of parameter space where concatenated ML fails. If individual gene trees are biased by concatenation, then so too will be the rooted triplets and unrooted quartets extracted from them for use with summary methods.

Regardless of the criticisms of published simulations, researchers in favor of summary gene tree methods face an apparent paradox: if typical protein-coding genes are immune to the effects of recombination, ILS, and concatenation (and can therefore be used to construct gene trees), then why not concatenate all the loci? Unfortunately, no theory yet exists that bounds the amount of recombination and ILS allowable while still producing correct trees. Further work is clearly needed to know how far such methods can be pushed.

5.2 Conditional concatenation and binning

Summary methods are problematic because ML is biased. We have now seen two causes for this bias: sequences that are too short to accurately model the substitution process (section 3.3) and sequences that are so long that they contain multiple conflicting topologies within them (section 5.1). In the next section we discuss “full” likelihood methods that can possibly
deal with the latter problem, and here we address strategies that have been used to increase the length of loci used as input to summary methods.

One strategy is that of conditional concatenation, in which loci are combined for analysis only if they pass some kind of test of concordance. Conditional concatenation has a long and well-cited history, and featured in the total-evidence versus consensus debate (e.g. Bull et al., 1993; De Queiroz, 1993). There are many different topology-based congruence tests available (see Leigh et al. 2011 for a comprehensive review), but many pitfalls to these approaches as well. As we mentioned earlier, there are potential biases in the bootstrap support for gene trees in the MSC: the edges in gene trees that are discordant with the species tree are likely to be short, with length determined by within-population coalescence. Hence bootstrap support for discordant trees may well be systematically lower than for concordant trees. Ironically, discordant trees will then be more likely to be combined with other trees than concordant trees.

More seriously, there is a fundamental problem with using topology-based tests to determine whether alignments can be combined: the main reason we are considering concatenation in the first place is because we cannot reliably construct phylogenies for single genes. How then can we expect these inferred single-gene trees to reliably inform us about phylogenetic dependencies? It may well be that character-based tests of incongruence can sidestep this issue, or at least appear to. However, since the objective is to emulate a non-recombining locus, it may well be more appropriate to apply one of the dozens of tests for recombination instead (e.g. Martin et al., 2010).

There has also been a lot of confusion in the literature about the interpretation of these combined genes. What is clear is that the combined genes should in no way be considered to be a linked, non-recombining locus with respect to the MSC. Indeed, there is no guarantee that the combined genes are even on the same chromosome. One possible interpretation of these conditionally concatenated loci is that they evolved separately along the species tree, but happen to come from gene trees that are not significantly different. After all, different gene trees are independent only conditioned on the species tree, and it is therefore no surprise that unlinked loci might have similar gene trees. By concatenating the genes, we are taking advantage of the fact that the gene trees are from the same species tree and so are interdependent, allowing us to fit more sophisticated and robust models. Once that inference process has completed, the genes should be considered independent with respect to the MSC—they just happened to have their gene trees estimated at the same time. This separation is implicit in “weighted statistical binning” (Bayzid et al., 2015). The combination of a sophisticated phylogenetic concordance test with a strategy like weighted statistical binning may offer a compromise choice that hits the “goldilocks” zone for multi-gene inference, though there is still plenty of work to do in understanding the systematic biases this could introduce.

6 Beyond summary methods versus concatenation

6.1 Full phylogenetic methods incorporating ILS

Summary gene tree methods are not the only way to incorporate gene tree heterogeneity into phylogenetic inference, and almost certainly are not the best way (though they are fast). Several methods exist that can carry out exact likelihood (or similar) calculations under the MSC, using these calculations to infer species trees from data (see Chapter 3.3 [Rannala et al. 2020]). Although these methods overcome many of the problems associated with summary approaches, they still face some of the same issues, especially those associated
The most widely used methods can be conveniently separated into two groups: those that use blocks of sequence, but assume no recombination within loci and free recombination between loci, and those that use only variable sites, but assume that there is free recombination between them. In the first category are methods implemented in BPP (Rannala and Yang, 2017), StarBeast (Heled and Drummond, 2009; Ogilvie et al., 2017) and PhyloNet (Wen et al., 2018). All three methods can work directly from individual gene alignments, calculating the likelihood of the data under the MSC. They accommodate sampling error in the gene trees that summary tree methods ignore. These methods (or their extensions; Zhang et al., 2018) are also able to infer species networks—essentially the species tree with reticulations—though the methods that do so require time-consuming MCMC sampling. Regardless of the way in which tree space is explored, these methods still assume that each input gene is a non-recombining unit, and therefore face some of the same modelling questions as summary methods.

In the second category are methods that assume free recombination between individually varying sites. Methods that use this type of data are varied, including SNAPP (Bryant et al., 2012), PoMo (De Maio et al., 2015), and SVDquartets (Chifman and Kubatko, 2014). While SNAPP and PoMo are optimal for species with recent splits (and multiple individuals sampled per species), SVDquartets is able to infer species trees with deep splits. These methods all avoid the issues with short non-recombining blocks of sequence, completely circumventing the problem by combining together a large number of independently evolving loci. Although complex substitution models incorporating all the different kinds of rate variation observed are not yet included in the tools listed here, these methods are some of the most promising for the future of phylogenetics.

6.2 Why genes should still be analysed separately

Even if you believe that species tree inference should only be carried out with concatenated data, it is still useful to infer trees for each gene. Arguments in favor of the examination of individual gene trees go back as far as the consensus/total-evidence debate (e.g. De Queiroz, 1993), and genomic data has only made this more true. Individual gene trees can reveal an enormous amount about variation in history along the genome, different rates of evolution in different genomic compartments, and different potential biases or patterns in a dataset. The signal in the data is in the variable history among loci, not just species relationships (Bravo et al., 2019).

One obvious example of where the study of individual gene trees can help is in cases of horizontal gene transfer (HGT) or introgression between species. The disagreement among trees is widely considered to be the best evidence for transfer (Soucy et al., 2015). Similarly, gene flow between sexually reproducing species can result in gene tree discordance at introgressed loci. The distribution of discordant trees along the genome is one of the few indications that introgression is occurring (e.g. Liu et al., 2014a), and the distinct heights and branch lengths of introgressed trees can help to disentangle complex histories (e.g. Fontaine et al., 2015; Kearns et al., 2018).

Because genes underlie traits, gene trees may also be a much better guide to trait evolution than species trees, especially when there is a lot of discordance (Hahn and Nakhleh, 2016). In cases with extreme levels of discordance, such as adaptive radiations, it may even be possible to associate individual discordant loci with incongruent traits (e.g. Pease et al., 2016; Wu et al., 2018). Radiations may be one of the best arguments for approaches that examine individual gene trees, as it becomes highly unlikely that any locus follows the inferred
Finally, the visualization of gene tree heterogeneity may itself be a worthwhile endeavor. Hillis et al. (2005) showed how a collection of inferred gene trees could be visualized in tree space using multidimensional scaling. Duchêne et al. (2017) used this multidimensional scaling approach to identify clusters of gene tree topologies supporting conflicting resolutions of the species tree, and were able to show that the clusters were generated by ILS. Other sorts of visualization tools may be equally useful in different contexts (e.g. Esser et al., 2004).

6.3 Moving forward

Phylogenetic inference is a hard problem, especially for deep divergences. As we have seen, much of the difficulty stems from how and what to model, and the extent to which different models impact on our inference.

Therefore, the choice of methods to use should be informed by the largest sources of error. At shallower timescales gene trees can be accurately inferred and ILS (and introgression) can be large sources of variance among gene trees. At deeper timescales the sources of variance flip, such that ILS becomes relatively less important. ILS certainly occurs at deep timescales, but many other processes also come into play, making the inference of individual gene trees much harder. While we hope that researchers interested in resolving relationships at, for instance, the base of animals keep the possibility of gene tree discordance in mind, it is certainly understandable that the methods they employ to infer a species tree do not model this process explicitly.

References


REFERENCES


3.4:20 REFERENCES


3.4.22 REFERENCES


